# SELF-CONSISTENT EVOLUTION OF RING GALAXIES

P. Mazzei<sup>1</sup>, A. Curir<sup>2</sup>, C. Bonoli<sup>1</sup>

- <sup>1</sup> Osservatorio Astronomico, Padova, Italy.
- <sup>2</sup> Osservatorio Astronomico, Torino, Italy.

#### Abstract

Ring galaxies are commonly known as objects where a burst of star formation was triggered by a close encounter with an intruder, maybe a satellite galaxy. BVRI CCD observations of five ring galaxies have been performed. Here we present the results of a self-consistent approach to reproduce their observed morphology and spectral energy distribution using updated N-body simulations and evolutionary population synthesis models extending from UV to far–IR wavelengths. Some suggestions about the evolutionary properties of these starburst galaxies are then derived.

## 1 Introduction

Although it is well known that many of the most spectacular examples of starburst galaxies are members of interacting systems, relatively little quantitative information is available on the overall effects of interactions on the star formation properties of galaxies. The degree of enhancement in the star formation rate varies over a large range, from galaxies which appear unaltered after the interaction, at least when observed at present, to galaxies with star formation bursts 10–100 times stronger than typically observed in isolated systems (Kennicutt, 1987; Schweitzer, 1990). This diversified scenario is a consequence of the large spread both in the phenomenologic conditions of the interaction and in the dynamical and physical stage of the colliding galaxies. In this paper, the first of a series on interacting systems, we focus our attention on a special class of interacting objects, the ring galaxies. They are commonly known as spiral galaxies where a burst of star formation is triggered by a close encounter with an intruder. An almost normal penetration of the companion at radii up to 40% of the disc radius (Lynds and Toomre, 1976; Appleton and James, 1990) generates their strange morphology where a more or less sharp ring surrounds an off-centered nucleus or an empty region. Following Appleton and Struck-Marcell (1987), rings provide both an understanding example of extended coherent starbursts and a laboratory study

of the effects of large–scale, nonlinear density waves on the interstellar gas. Ring galaxies are strong far–IR emitters. Their IR luminosity,  $L_{FIR}$ , as derived by IRAS colors, is even larger than for barred galaxies (De Jong et al. 1984), the most luminous class of the Shapley–Ames galaxies. Furthermore, some rings show  $L_{FIR}$  also greater than the prototype starburst galaxy M82. As a general rule such galaxies emit as much infrared radiation as blue optical light. Therefore the mean value of the blue absolute luminosity,  $L_B$ , of ring galaxies agrees well with that derived by Keel et al. (1985) for Arp galaxies in general,  $L_B = 1.4 \times 10^{10} (100/H_0)^2 L_{\odot}$ , about 0.5 mag brighter than for non interacting ones.

In this paper we adopt a self–consistent approach to analyze the morphological and photometric properties of five of them through up-to-date N–body simulations and evolutionary population synthesis (EPS) models extending from UV to far-IR wavelengths. We use the results of our N–body simulations as inputs of our EPS models to derive some suggestions for modeling the burst and the luminosities involved in the optical and far-IR ranges. Models obtained with a more refined SPH code including an internally consistent luminosity evolution will be treated in a later paper.

Galaxies were selected from the list of Appleton and Struck-Marcell (1987) and observed by one of us (C.B.) in BVRI. Far-IR (FIR) data come from IRAS catalogue (Version 2).

The plan of this paper is the following: observations and data reduction are described in Section 2; corresponding simulations of collisions between stellar disks and intruders are in Section 3 while in Section 4 we discuss the treatment of the burst; some suggestion is also given about the postburst evolution of such systems. In section 5 we point out the results for each galaxy and in section 6 our conclusions.

## 2 Observations

BVRI CCD observations were carried out at Padova-Asiago Observatory using a GEC CCD with a pixel size of  $22 \,\mu\text{m}$ , corresponding to 0.28 arcsec. Exposure times were 50m in B, 30m in V, 20m both in R and in I. Typical seeing during the observing run was 1".8

The reduction of images consisted in the standard procedure. Pedestal was removed by subtracting a mean value estimated in the overscan region of each frame. Several flat field exposures, obtained at twilight in each color, were used to remove variations in pixel-to-pixel sensitivity. Sky background was determined in each frame by making a histogram of the gray levels of all pixels and locating the peak value by a gaussian fit of the low level part.

Frames of selected stars in M67 cluster, obtained in the same nights, were used to calibrate the instrument photometry. The uncertainty in zero points, estimated from the statistics of the calibration constants derived from different standard stars in the same frame, turns out to be some hundredth of magnitudes in the four bands. However, we observed several standard fields in different nights and we found that the measurements show rms  $\simeq 0.08$ . This increased

uncertainty is likely due to the extinction corrections and changes in photometric conditions from night to night. All in all we have assumed that the calibration error is  $\simeq 0.1$  mag.

Figure 1 shows R-band isophotes of our sample and Table 1 presents their BVRI magnitudes in the Johnson (1966) system.

## 3 N-body simulations

We performed numerical simulations of collisions between stellar disks embedded in static halos and suitable intruders. The code used is the Hernquist (1987) TREECODE which employs a tree structure to calculate the gravitational forces; this is a useful tool to study systems endowed with strong deviation from spherical symmetry (Curir, Diaferio, De Felice, 1993).

The disks have been relaxed down by solving numerically the Laplace equation in cylindrical coordinates (Binney and Tremaine, 1987) and then immersed in a massive halo structure (King model, 1981). The system is evolved by several rotation periods to test its stability. The halo has an important heating effect on the disk during the assessment.

The companions used as intruders are massive points or King spheres of different radii. A series of central collisions have been performed varying the angle of incidence and the velocity of the companion. We refer for a more complete and detailed set of simulations in the space of the numerical parameters to Curir and Filippi (1994). Here we present only the more suitable simulations in order to match the morphologies of our selected ring galaxies. From the pioneering numerical work of Lynds and Toomre (1976) many papers are now available in literature on this subject. Recently, models have been produced using non dissipative N-body codes by Luban-Lotan and Struck-Marcell (1989) and by Huang and Stewart (1988); furthermore, gas dynamics and dissipative phenomena have been taken into account by Gerber, Lamb and Balsara (1992), Weil and Hernquist (1993), Struck-Marcell and Higdon (1993) which presented very refined simulations, devoted to provide good models for specific astronomical objects. In this paper we will use pure N-body simulations to have more insights in the role played by the dynamical friction in the case of the intrusion.

In the following we briefly summarize the main points of our model and then present our results.

#### 3.1 The numerical methods

In the system of units employed, G = 1 (G is the gravitational constant), the length scale is 0.1, and the mass of the disk and of the intruder are equal to 1. Translating these values into physical units, mass unit is  $5 \times 10^{10} M_{\odot}$ , distance unit is 20 Kpc and time unit is 158 Myr. A fixed timestep  $\delta t$  of 0.01 (time units) is used to update the gravitational forces which are calculated including quadrupole moments of the gravitational potential and using a tolerance parameter  $\theta = 0.8$ .

The target galaxy model consists of two components: a spherical halo and an exponential disk. The mass ratio of these components is 4:1. The model mass-points, tracers of the true mass distribution, interact through the pseudo-keplerian softened potential

$$\Phi = \frac{-K}{(r^2 + \varepsilon^2)^{\frac{1}{2}}}$$

where  $\varepsilon$  is the softening parameter adopted to avoid extremely close encounters.

The softening length for the gravitational force depends on the type of particle: the stellar particles employ a softening  $\varepsilon_d = 0.015$  while halo particles employ  $\varepsilon_h = 0.038$ 

The disk and the halo are modeled as N-body systems with 3192 and 5000 particles respectively; these numbers are large enough to adequately describe their kinematical properties (see Curir and Filippi, 1994 for a detailed discussion on this subject). The halo particles are assumed to be dark matter particles. The higher value of the softening length accounts for the weakly interacting nature supposed for these particles.

The spatial distribution of stars in the disk follows the planar surface density law

$$\rho(r) = \rho_0 e^{-\frac{r}{r_0}}$$

where  $r_0$  is the disk scale length, equal to 2 Kpc.

To obtain each particle position-vector according to the assumed mass density we use the technique called "rejection method" for generating random deviates whose distribution function is known (Press, Flannery, Teukolsky, Vetterling 1990). Azimuthal angles are chosen randomly in the interval  $[0, 2\pi]$ . The vertical co-ordinate is extracted from a gaussian distribution with a dispersion equal to the 1% of the disk scale length.

We assign circular velocities analytically to disk stars, through a particular expression of the potential obtained with the aid of Jeans equations. Radial, azimuthal and vertical velocity dispersions are locally imposed on disk particles; dispersions are monitored through a Toomrelike parameter Q (Toomre, 1978), a true "stability thermometer" for the exponential disk (in our simulations Q=1.6). The Q parameter is an input parameter for our model.

The halo is a King model with the characteristic distribution function:

$$f_K(E) = \begin{cases} \rho(2\pi\sigma^2)^{-\frac{3}{2}} (e^{\frac{E}{\sigma^2}} - 1) & \text{when } E > 0; \\ 0 & \text{when } E \le 0. \end{cases}$$

where  $\rho$  is the density,  $\sigma$  the velocity dispersion and E the total energy of the stars. King models are dynamically stable systems.

To obtain the final galaxy model we relax the two components together, superimposing the gravitational potential generated from the exponential disk on the potential of the spherical model which has a radius twice that of the disk. The two components have approximately

the same mass within three disk scale lengths. The resulting rotation curve of the disk is substantially flat and reproduces quite faithfully the observed curves for a large part of spiral galaxies (Rubin et al. 1980) (see for ex. Fig.5a, curve labeled 1).

The energy is conserved better than 0.5% and the angular momentum better than 0.01% over the whole run for each model.

#### 3.2 Results

We performed several simulations using different intruders and a different impact velocity, defined as the initial relative velocity between the two systems placed 60 Kpc a part. The mass of the intruder is equal to the mass of the disk target (Curir and Filippi, 1994). In our space of parameters we singled out two different regimes for the ring formation.

In the first the impact velocity is below the escape velocity of the intruder (i.e  $\leq 200 \, km \, s^{-1}$ ) and the dynamical friction dominates the interaction. As the intruder oscillates crossing the disc, it loses a remarkable part of its mass and is finally trapped in the system. The time evolution of the ring structure is 190 millions of years depending slowly on the radius and on the velocity of the companion. The ring appears while the intruder is very near to the disk, between 0. and 1.5 in our numerical units (i.e. 30 kpc). Fig.s 2a and 2b show an edge—on and the face—on view respectively of the predicted evolution for a typical case in this regime. This is characterized by a range of velocities from  $\approx 100 \, km \, s^{-1}$  to  $200 \, km \, s^{-1}$ .

In the second regime the dynamical friction is much less active because the impact velocity is higher than the escape velocity, thus the ring is formed when the intruder is already far from the disk, at a distance between 1.5 (30 Kpc) and 3 (60 Kpc). The intruder emerges from the collision very unperturbed in shape but enlarged in volume (on the average the radius is almost twice the original one). The time evolution of the ring structure is 112 millions of years when the radius of the intruder is equal to 0.28 of the target disk. As far as these high impact velocity cases are concerned, Fig. 3 shows the time evolution of the morphology related to the formation of a ring with a nucleus (the projected intruder) whereas Fig. 4 describes an empty ring. In Fig. 5 the behaviour of the rotation velocity (panel a)) and that of its components in the polar (panel b)) and in the radial (panel c)) directions, for the same simulation as in Fig. 3, is presented; in particular the curve number 4 of panel b) shows that the disk relaxes to a relatively high value of the z component of the velocity dispersion after the merger.

By performing different simulations using smaller and smaller companions we obtain practically the same configurations provided we scale the impact velocity in a linear way with the radius of the intruder. However, the time of evolution,  $\Delta t$ , lengthens. In particular a simple material point behaves in this regime like a King sphere (of unit radius) giving rise to a ring living 350 millions of years. So in this regime we can define the stage of the evolution with a parameter  $\tau$ , given by the ratio  $t/\Delta t$  where t starts with the beginning of the interaction. We point out that the two regimes previously outlined are not the only two possibilities for the

formation of a ring galaxy. There are more other possibilities which we did not analyze, like the disruption of a small companion, which may generate an empty ring, or the presence of a nucleus inside the ring simply because the target retains its bulge, which we did not include in our simulations; Curir and Filippi (1994) explored also the possibility of a "spontaneous" ring formation. In particular these regimes, driven by the dynamical friction, are simply related to the mechanism of the intrusion and provide good numerical fits for our sample of ring galaxies.

The related density waves have quite different features (Fig. 6). In the first one the wave appears as a light perturbation of a decreasing exponential density distribution. At the same time its amplitude is much less than in the second regime, where the wave expands starting from a strong central depression. In both cases, well after the collision, the system settles into an exponential disk thicker and larger than the original one.

## 4 Photometric properties from UV to far-IR: their evolution

Our EPS models, accounting for the dust effects, provide chemical and photometric evolution of a galaxy in a self-consistent way from ultraviolet up to 1 mm. The synthetic SED incorporates stellar emission, internal extinction and re-emission by dust. The stellar contribution, including all evolutionary phases of stars born with different metallicities, extends as far as  $25\,\mu\text{m}$ . Dust includes a cold component, heated by the general radiation field, and a warm component associated with HII regions. Observations in the short wavelength range  $(25–60\,\mu\text{m})$  and in the long one (over  $60\,\mu\text{m}$ ) allow us to disentangle their luminosities and temperatures.

Emission from policyclic aromatic hydrocarbon molecules (PAH) and from circumstellar dust shells are also taken into account; the bulk of their emission centers in the 5-25  $\mu$ m spectral range, where the pure stellar contribution rapidly fades. This model has been successfully applied to spiral galaxies, with particular attention to our own galaxy (Mazzei,Xu and De Zotti, 1992, hereafter referred to as MXD92) and to early-type galaxies (Mazzei and De Zotti, 1994a; Mazzei, De Zotti and Xu, 1994). In Appendix we briefly summarize the main points of this model.

We remember that the star formation rate (SFR) and the initial mass function (IMF) together with its lower and upper mass limits, are important model parameters. As discussed in MXD92, the metallicity of the system depends on their values and, as a consequence, the far–IR output. Since we are modeling in a self–consistent way both the chemical and the luminosity evolution, given the IMF with its mass limits, the metal enrichment and the residual gas fraction, as well as the total far–IR luminosity, are only dependent on the assumed SFR. This is slowly decreasing before the interaction since, due to the previous discussion (Sect. 3), a disk-like configuration well represents the unperturbed state. However a central collision with a spherical intruder may trigger a strong burst of star formation (Kennicutt et al. 1987; Wright et al. 1988) lasting some 10<sup>8</sup> years (see Sect. 3). Therefore we modify the normal disk evolution (see also MXD92)

superimposing a strong burst of star formation at an age of 12 Gyr.

In this paper we present the first effort to analyze the effect of a burst on the overall SED of galaxies.

We follow a general approach without any change of the IMF as well as of its lower and upper mass limits during the burst; this burst is defined by its length and intensity, which is the ratio between the value of the SFR at its beginning and that before its onset.

Fig. 7 shows the evolution of the gas fraction, metallicity and optical depth (see Appendix), for models computed with different  $m_l$  values and different burst length; in Fig. 8 their color evolution is presented. We emphasize that the burst affects only the UV–near IR SED whereas the far–IR output, depending on the optical depth, is a strong function of  $m_l$  value alone.

The shape of the far–IR SED depends on three parameters,  $I_0$ ,  $I_w$  and  $R_{w/c}$  which represent the maximum value of the interstellar radiation field, the average energy density inside HII regions and the warm to cold luminosity ratio respectively (see Appendix for more details). These are completely defined by fitting the observed IRAS colors. Given the strong internal consistency of our model, there is not a large range of possible values for these far–IR parameters. As we will discuss with more detail, we suggest two different combinations of such parameters able to match the overall SED of a galaxy, consistent with our N–body simulations. In the following, in fact we discuss the treatment of the burst and the post–burst phases as derived from a self-consistent approach with N–body simulations. These simulations provide the length of the burst and the spatial distribution of the matter inside our galaxies. The first one allows us to reduce the number of free parameters needed to describe the burst whereas the second one gives us some information on the physical conditions which are affecting the shape of the far–IR SED.

#### 4.1 The burst phase.

As far as optical and near-IR data only are concerned, the parameters defining the burst, its length and intensity, are not independent but inversely correlated. UV data, coupled with near–IR observations, could in principle disentangle their effects. In fact, the stronger the burst, the larger the number of massive stars, i.e. the UV flux, provided from the burst. Thus blue UV-V, V-K colors could suggest stronger and younger bursts, whereas red colors weaker and older stages since red supergiant stars, AGB stars, will appear  $\approx 10^8 \, yr$  after its beginning. Nevertheless the extinction smooths away these differences making the galaxies appear older and the SFR lower. However the energy absorbed by dust in the short wavelength region of the galaxy spectrum must be re-emitted in the long wavelength range. So in principle the observed overall SED entails all the information necessary to understand the behaviour of the SFR and to define the model parameters.

Ring galaxies are very faint objects so UV observations are extremely rare; only VV787 has been observed (Schultz et al. 1991) and its data suffer from aperture correction problem.

It is well known, however, that the far–IR output of starburst galaxies is higher than that of normal galaxies (Soifer et al. 1987; Joseph, 1990). Ring galaxies also match these expectations (Appleton and Struck-Marcell, 1987): some of them in fact have been detected by IRAS satellite. Our model, by means of its large spectral coverage, provides a useful tool for understanding the available data.

Taking into account the previous considerations we use the time spent in the interaction suggested by N-body simulations as an input for our EPS model: the lifetime of the ring defines the length of the burst. Results are not strongly dependent on the behaviour of the SFR during the burst if we adopt a SFR constant or quickly decreasing with the same dependence on the mass of gas as before the onset of the burst.

Given the burst parameters, the far–IR output of the system strongly depends on the lower mass limit of the Salpeter IMF,  $m_l$ . The higher the value of  $m_l$ , the larger the number of massive stars formed and then the larger the amount of heavy elements provided; this enhances the extinction effects simply because the optical depth of the system is proportional to the metallicity of the gas (see Fig. 7). We find that  $0.01 \le m_l \, (m\odot) \le 0.20$  accounts for the overall properties of our sample. Most ring galaxies are characterized by  $L_{FIR}/L_B$  ratios larger than those of normal galaxies (Appleton and Struck-Marcell, 1987), thus, following from our approach the largest  $L_{FIR}/L_B$  ratios suggest the highest  $m_l$  values. It is not surprising that ring far–IR SED may be different from that of a "normal" spiral since the interaction greatly perturbs the initial distributions of gas and dust.

We attempt an approach consistent with the results of N-body simulations (cfr. 3.2) also in modeling the far-IR emission, in particular as far as the cold dust distribution is concerned. The luminosity of such a component, in fact, depends on the interstellar radiation field distribution through the parameter  $I_0$ , i.e. the central value of its energy density (see Appendix for more details). As discussed in the previous section, strong collisions provide high compression and concentration of matter in the density wave. At the same time a large depletion of gas and stars is produced in the central regions. According to Leisawitz and Hauser (1988), a remarkable fraction of the luminosity of OB stars can escape from Galactic HII regions and may contribute to the diffuse energy density, depending both on the efficiency of the star formation and on the optical depth in the ring (i.e. on the local gas properties in the star forming regions). So, we expect to match the IRAS colors using two very different values of the maximum radiation field,  $I_{r_0}$  or  $I_0$ . We define  $I_{r_0}$  as the maximum amplitude of a diffuse radiation field centered in the density wave instead of in the disk  $(I_0)$ . Following the previous discussion we expect the values of  $I_{r_0}$  larger than  $I_0$ . These possibilities, i.e. the " $I_{r_0}$ " and the " $I_0$ " cases, correspond to different physical conditions inside the galaxies, of course, in particular different amount of cold dust, lower for  $I_{r_0}$  than for  $I_0$ , is provided.

Given the strong internal consistency of our model, different combinations of the far–IR parameters with the same  $m_l$  value are ruled out. The far–IR distribution extends, indeed, to the longer wavelength the lower  $I_0$  or  $I_{r_0}$ . Therefore, for an observed  $L_{100}/L_B$  ratio, the

decrease of  $I_0$  or  $I_{r_0}$ , for example, requires higher far–IR output to match the same IRAS colors, thus higher  $m_l$  values. In the following section we will present with more details the results of models describing two extreme physical situations compatible both with observations and with our simulations i.e. the warmest  $I_{r_0}$  value, corresponding to the lowest  $m_l$ , and the coldest  $I_0$  case, corresponding to the largest  $m_l$  (see Tables 2 and 3). Models computed with  $m_l$  values inside this range could also match the data; in particular rising  $m_l$  we have to lower  $I_{r_0}$ , to increase the  $R_{w/c}$  ratio and may be to decrease the  $I_w$  value.

As suggested from Fig. 9, submillimeter observations could provide a useful test to discriminate between these possibilities keeping in suspence the global far–IR luminosity and the dust content of such galaxies.

In Fig. 9 the overall synthetic SEDs for our sample galaxies are presented; heavy lines show the fit obtained in the " $I_{r_0}$  case" (see Table 2) light lines are in the opposite one, corresponding to the " $I_0$  case" (Table 3); for comparison, MXD92 found for the maximum radiation field a value of  $7I_{local}$ .

#### 4.2 The postburst phase.

For deriving the overall synthetic SED some assumption concerning the shape of the far–IR SED (i.e. the values of the far–IR parameters) is needed, since at this stage no observational constraints on the far–IR appearance of postburst galaxies, faint FIR emitters, of course, are available. Therefore we can extract some useful information from our N–body simulations. A more realistic picture will be available in the next future with the help both of more sensitive far–IR measures, provided from ISO satellite, for example, and of high resolution images (HST) of galaxies. These would be compared with the results of dynamical (N–body) and hydrodynamical simulations (SPH) to identify the post–burst cases.

As discussed in Sect. 3.2 the final effect of the interaction is to decrease the diffuse energy density with respect to its initial unperturbed value since the system re-arranges in a disk thicker and larger than the unperturbed one. We come to the same conclusion with an independent approach, i.e. taking into account that the number of OB stars, which may strongly contribute to the diffuse radiation field, in particular in the " $I_{r_0}$  case", rapidly fades after the burst. Thus we can estimate the far–IR emission of the cold dust component, long after the burst, simply assuming  $I_{r_0}$  proportional to some power,  $\alpha$ , of the number of OB stars. Models suggest  $\alpha \approx 0.3 - 0.4$ . The same suggestion cannot apply, of course, in the " $I_0$  case" since OB stars do not provide a substantial feeding to diffuse radiation field which is, in fact, much lower than in "normal" disks also during the burst. The luminosity of the warm emission can be derived assuming the same  $R_{w/c}$  ratio as for nearby spirals ( $R_{w/c} = 0.43$ , Xu and De Zotti, 1989),  $I_w$  being the same as during the burst.

Fig.s 10(a-e) compare the SEDs of our galaxy sample during the burst (heavy line) and 2 Gyr after the burst (light line) i.e at an age of 14 Gyr.

 $L_{FIR}/L_B$  ratios as large as 10 times the galactic one (of about 2.5, MXD92) can arise also at 14 Gyr, depending on the  $m_l$  value needed to fit the observed data.

The bolometric luminosity of the systems is approximately 10 times lower than during the burst and the B luminosity drops by a factor of approximately 10–30. Thus these systems will appear as very faint red galaxies. The past interaction, which exhausted a significant amount of the residual gas (35% – 60% for our sample), reflects in the optical region of the spectrum through very red B-V colors, redder than the average ones of normal disk galaxies. At 14 Gyr models suggest  $0.85 \le B-V \le 1$ , where the reddest colors correspond to the greatest  $m_l$  values (see also Fig. 8). At the same age V-K ranges from about 3.5 mag, as for an unperturbed disk, up to 4 mag and 4.4 mag for  $m_l = 0.01$ , 0.05 and 0.1  $m_{\odot}$  respectively. Only NGC 2793 deviates from this behaviour. Its colors are not strongly affected since the burst consumed a low fraction of the residual gas. Therefore this appears as a very blue galaxy, its colors being typical of an Irr also long after the burst.

## 5 Comparison with observations

In this section we attempt a careful description of the morphological and photometric properties of each galaxy in our sample. The observed isophotes (Fig. 1) will be compared with suitable isodensity contour levels obtained from N-body simulations (Fig. 11). The particle positions of the numerical dump chosen to represent the galaxy are projected on a suitable plane and then the isodensity contours are extracted. The time spent in the interaction, as described in Section 3, is the length of the burst for our EPS model. This length is practically constant in the first regime (0.2 Gyr) whereas it depends on the radius and on the velocity of the intruder in the second one, ranging between 0.11 and 0.35 Gyr. In this case we refer our fits to the parameter  $\tau$  which settles the stage of the burst evolution (see Sect. 3.2); it is not surprising that such a stage may be different for our morphological and photometric fits although, in most the cases, they well agree.

We derive some suggestions about the strength of the collision or the intensity of the burst (as defined in Sect. 4), the total far–IR emission,  $L_{FIR_{tot}}$ , the absolute bolometric luminosity,  $L_{bol}$ , and the residual amount of gas.

We point out that, given the internal consistency of UV-optical and far-IR SEDs, widely discussed both in Sect. 4 and in the Appendix, only two combinations of  $m_l$  and far-IR parameters for each galaxy are discussed. These correspond to the extreme " $I_{r_0}$ " and " $I_0$ " cases respectively (see Table 2 and 3). Submillimeter observations should be careful recommended to reduce the uncertainty further.

For reasons of comparison we summarize here some useful values derived for our own galaxy (MXD92):  $L_{FIR_{tot}}/L_B = 2.6$ , the corresponding ratio using  $L_{FIR}$ , i.e. the far–IR luminosity computed from 42.5 to 122.5  $\mu m$ ,  $L_{FIR}/L_B \simeq 0.9$ , and  $L_{FIR_{tot}}/L_{bol} \simeq 0.3$ .

We used  $H_0 = 50 \, km \, s^{-1} \, Mpc^{-1}$ .

#### 5.1 VV 789 or I Zw 45

The isodensity contour levels are represented in Fig 11a (for comparison see Fig. 1). The simulation describes a collision of a disk target lying in the xy plane with a King sphere having a radius 0.28 times that of the disk. The velocity of the intruder is  $1140 \, km \, s^{-1}$  inclined of 30 deg with respect to the normal, z, to the disk plane. The stage of the ring evolution correspond to  $\tau = 0.20$ .

The overall SED of this galaxy (Fig. 9(a)) requires a strong burst of star formation whose intensity being 60. Models have been computed with a lower mass limit of 0.01 and 0.05  $m\odot$  respectively for the  $I_{r_0}$  (heavy line) and  $I_0$  (light line) cases. Our photometric fits correspond to different stages of the burst evolution,  $\tau = 0.75$  and 0.28 respectively.

The residual fractions of gas are 0.25 and 0.54 corresponding to  $(1.56-4.07) \times 10^{10}~m\odot$  respectively. For comparison Theys and Spiegel (1976) derived a value of  $4.56 \times 10^9~m\odot$  for the mass of neutral hydrogen alone. The system appears as a very blue luminous galaxy with  $L_B (10^{10}~L\odot) \simeq 4.5$  (Appleton and Struck–Marcell, 1989).

From the previous models we derive  $L_{FIR_{tot}}/L_{bol}=0.50$  and 0.68, which correspond to  $L_{FIR}/L_{bol}=0.44$  and 0.21. Moreover we predict  $L_{FIR}/L_{B}=2.01$  and 1.59 respectively; the last ratio increases to 2.3 and 5.24 taking into account the total far–IR luminosity; thus the long wavelength range could include up to 70% of the far–IR luminosity of this galaxy.

### 5.2 VV 330 or UGC 5600

The parameters describing the intrusion are the same as for VV798 but the collision velocity is orthogonal to the plane of the disk. In Fig. 11b we present the morphology of the simulated system (for comparison see Fig. 1) taken to a stage  $\tau = 0.40$  of the ring evolution.

Two extreme models, computed with different lower mass limit of the IMF, 0.05 and 0.1  $m\odot$  respectively, but the same intensity of the burst, 60, match quite well the optical SED and the IRAS colors of this galaxy (Fig. 9b). Our fits correspond to similar stages of the interaction,  $\tau = 0.43$  and 0.29 respectively, where the residual fractions of gas are 0.47 and 0.63 respectively. We derive a blue luminosity,  $L_B = 0.56 \times 10^{10} L\odot$ , like normal galaxies (De Jong et a. 1984) however a large fraction of the bolometric luminosity comes out at the longest wavelengths:  $L_{FIR_{tot}}/L_{bol} = 0.70$ –0.68 respectively. We find  $L_{FIR}/L_B = 3.12$  and 3.04 which rise up to 6.0 and 7.6 including the total far–IR luminosity.

## 5.3 VV 787 or Arp 147

In a recent paper Gerber et al. (1992) showed that this galaxy is the result of an off-center high velocity collision between an elliptical and a spiral of the same mass, the intrusion being perpendicular to a disk plane. This corresponds to a collision with a impact velocity  $v \simeq 450 \, km \, s^{-1}$  which well agrees with our second velocity regime. Their isophote fit suggests  $tau \simeq 0.66$  from the beginning. For the same stage, models matching quite well the optical SED and the IRAS colors of this galaxy (Fig. 9c) correspond to  $m_l$  0.05 and 0.1  $m\odot$  with intensities of 60 and 100 respectively.

The blue luminosity of this galaxy is about  $1.4 \times 10^{10} L\odot$  (Appleton and Struck–Marcell, 1989). We derive large  $L_{FIR_{tot}}/L_{bol}$  ratios, 0.71–0.78 respectively,  $L_{FIR}/L_{B}$  ratios of 2.70 and 4.50, increasing of the same factor, 2.5, including the global far–IR emission, and residual fractions of gas of 0.54 and 0.43 respectively.

#### 5.4 VV 32 or Arp 148

The impact velocity used for this simulation is  $160 \, kms^{-1}$ . Thus this ring is the only one in our sample described by the first velocity regime (see Sect. 3.2). This result confirms previous optical and mid-infrared analyses (Joy and Harvey, 1987) suggesting that this galaxy is coalescing. In Fig. 2 we have shown the time evolution of its morphology. Fig. 11c presents the corresponding isodensity curves (for comparison see Fig. 1).

The BVRI region of the SED of this galaxy has been analyzed by Bonoli (1987) and we refer to that paper for a detailed discussion. Models computed with the same burst intensity, i=60, and different mass limits,  $0.1\,m\odot$  and  $0.2\,m\odot$ , match well the overall SED of such a galaxy (Fig. 9d) for a warm and a cold diffuse radiation field respectively (see Tables 2 and 3). The  $L_{FIR}/L_B$  ratio suggested by these models, 6 and 7 respectively, one of the greatest in the sample of ring galaxies by Appleton and Struck-Marcell (1987), becomes 10 and 16 including the total far–IR luminosity. All models predict that dust absorbs about 80% of the bolometric luminosity of the galaxy. Even if the the burst consumes 75% and 40% of the residual mass of gas, the system retains a substantial gas fraction,  $\approx 0.4$  in both the cases which correspond to different stages of the burst evolution, 0.15 and 0.05 Gyr from its beginning respectively.

#### **5.5** NGC 2793

Its morphology is obtained with an head–on collision of a material point. The impact velocity is  $600\,kms^{-1}$ . The simulated isodensity contours of the system seen face–on are represented in Fig. 11d at  $\tau=0.17$ .

Our results suggest a very early stage in the burst development, in agreement with the short radius observed for this ring (Appleton and Struck-Marcell, 1989). Fig.9e, heavy line,

shows that the overall IRAS SED of this galaxy is matched well by a very high radiation field,  $I_{r_0} = 45I_{loc}$  (see Table 2), as before the onset of the active phase of star formation in the ring: stars compressed in the ring increase the diffuse energy density, starting then the new process of star formation triggered by shocks. Models suggest:  $L_{FIR}/L_{FIR_{tot}} = 0.5$  and  $L_{FIR}/L_{bol} = 0.22$ .

At the onset of the burst, whose intensity is 10, a low central radiation field and a large amount of warm dust, about 3 times larger than in our own galaxy, are required to match the data (see Table 3). The far-IR output is leaded by the warm component which entails 40% of the total IR luminosity of the galaxy instead of 17% as in our own galaxy; moreover, although the ratio  $L_{FIR}/L_{FIR_{tot}}$  is practically the same as in our own galaxy, 0.44 and 0.39 respectively, the warm emission encompasses about 65% of  $L_{FIR}$  instead of 14%. The  $L_{FIR}/L_{B}$  ratio is approximately double of the galactic one and  $L_{FIR_{tot}}/L_{B}$  is larger by a factor of 1.5. However  $L_{FIR_{tot}}/L_{bol} = 0.4$ , like the Galaxy.

The lower mass limit for the best fit model is 0.01  $m\odot$ .

We derive a large fraction of residual gas, 0.65, and an absolute blue luminosity  $L_B = 3.26 \times 10^9 L_{\odot}$ , the lowest in our sample. Our fit implies a very early stage in the burst evolution corresponding to  $\tau = 0.05$  and to a residual gas consumption of only 15%. The burst does not affect the colors of this system assuming the typical colors of an Irr during whole evolution.

#### 6 Conclusions

We investigate the evolution of ring galaxies in a self-consistent way using up-to-date N-body simulations and EPS models providing the overall SED, from  $0.06\,\mu\mathrm{m}$  up to 1 millimeter .

Results are compared with optical and far–IR data of a sample of 5 ring galaxies selected from the Appleton and Struck-Marcell (1987) list. Data from B to I bands are derived from our CCD photometry, far–IR data come from IRAS catalogue (Version 2).

Although in principle a ring galaxy can arise from a large number of situations (Curir and Filippi, 1994 and references therein), nevertheless the morphologies of our sample are well reproduced by a central collisions between a disk galaxy and a spherical intruder. We singled out two different behaviours for the ring formation, according with the collision velocity being below or up the escape velocity. In the former one the ring seems to be a transient structure, ending in a complete merging. VV32 is an example of this regime leaded by the effect of dynamical friction and lasting about  $0.2\,Gyr$ .

Stronger collisions, instead, producing an empty ring (i.e VV787), require higher impact velocities. In this second regime the length of the interaction depends on the radius of the colliding galaxy: the smaller the radius the longer the interaction providing a ring configuration. When the burst turns off, after  $0.1-0.35\,\mathrm{Gyr}$  from its beginning, a new disk-like configuration would arise owing to the re-arrangement of the galaxy. This will appear as a disk with a slightly larger radius and a higher thickness. These simulations give insights in the modeling the

photometric properties of these objects providing the length of the burst and the distribution of the matter inside our galaxies. The former one enables us to reduce the number of free parameters needed to describe the burst whereas the second one suggests two different extreme values of the radiation field inside these galaxies both matching well their IRAS colors. It is well known, indeed, that rings are powerful far-IR emitters (Appleton and Struck-Marcell, 1987), furthermore their dust temperature, as suggested by their  $f_{60}/f_{100}$  ratio, is also warmer than isolated disk galaxies. We find  $L_{FIR}/L_{bol}$  ratios exceeding those of normal galaxies by at least a factor of two, a value of 0.7 being typical. As suggested from our EPS models, the lower mass limit of the IMF is the most important parameter driving the far-IR output of these systems. Values up to 20 times greater than those derived to match the overall SED of our own galaxy ( $m_l = 0.01$ , MXD92) are needed. From our synthetic SEDs we derive  $L_{FIR_{tot}} \geq 2L_{FIR}$ being  $L_{FIR}$  computed from 42.5 and 122.5  $\mu$ m, with a warm luminosity which entails up to one half of the whole far-IR emission. Unfortunately their global far-IR luminosity is uncertain for the lack of submillimeter data. Observations in this spectral domain will provide useful information on the distribution and the amount of dust in such galaxies. Data in the mid-far IR would also provide important hints on the understanding the nature of the dust in ring galaxies. Observations with ISO satellite, whose launch is scheduled for september 19, 1995 will be performed. ISO instrumentation, with its high sensitivity, better than IRAS, and its larger spectral coverage, will allow us to better define the observed SEDs from 7 up to  $200 \,\mu\mathrm{m}$ . Observations in the short wavelength range will provide useful information on PAH and warm dust contributions, those in the long wavelength one, in particular at  $200 \,\mu\text{m}$ , will enable us to disentangle models with different maximum radiation field solving the puzzle of cold dust amount and distribution as so as of the true  $L_{FIR}$  luminosity.

After the burst the star formation rate lowers to a quasi-constant value driving a very slow system evolution. In time the bolometric luminosity of the galaxy will be dominated by a number of red giant stars larger than in normal galaxies. Systems which have experienced a ring phase characterized by a strong burst of star formation, would keep a larger  $L_{FIR}/L_B$  ratio than normal disk galaxies as a consequence of the past interaction. The predicted SED will reflect this situation showing very red optical near–IR colors and a slightly cooler far–IR emission following the re-arrangement of the disk. Systems which have experienced a burst of low intensity, like NGC 2793, will appear as very blue low brightness galaxies.

## 7 Appendix

In the following we summarize the fundamental assumptions of the model which allows us to derive the SED of galaxies over the whole frequency range, from UV ( $\lambda = 0.06~\mu\,m$ ) to far-IR ( $\lambda = 1000~\mu\,m$ ) (see MXD92 for more details).

#### 7.1 The chemical evolution model

We have adopted a Schmidt (1959) parametrization, wherein the star–formation rate (SFR),  $\psi(t)$ , is proportional to some power of the fractional mass of gas in the galaxy,  $f_g = m_{gas}/m_{gal}$ , assumed to be, initially, unity  $(m_{gal} = 10^{11} \ m\odot)$ .

$$\psi(t) = \psi_0 f_a^n \, m \odot \, yr^{-1}. \tag{1}$$

The initial mass function (IMF),  $\phi(m)$ , has a Salpeter (1955) form:

$$\phi(m)dm = A\left(\frac{m}{m\odot}\right)^{-2.35} d\left(\frac{m}{m\odot}\right) \qquad m_l \le m \le m_u, \tag{2}$$

with  $m_u = 100 \, m\odot$  and  $m_l \leq 0.2 \, m\odot$  (see text).

The influence of a different choice of the power law index, n, for the dependence of the SFR on the gas density has been discussed by Mazzei (1988). The effects of different choices for the IMF and its lower mass limit,  $m_l$ , are analysed in MXD92 for late—type systems and in Mazzei et al. (1994) for early—type galaxies. The general conclusion is that the overall evolution of late—type systems is weakly depending on n. We put n = 1 and  $\psi_0 = 4 m \odot /yr$ .

The galaxy is assumed to be a closed system with gas and stars well mixed and uniformly distributed. However, we do not assume that recycling is instantaneous, i.e. stellar lifetimes are taken into account.

The variations with galactic age of the fractional gas mass  $f_g(t)$  [and, through eq. (1), of the SFR,  $\psi(t)$ ] and of the gas metallicity  $Z_g(t)$  are obtained by numerically solving the standard equations for the chemical evolution.

As far as the burst is concerned we point out that the observed blue and red morphologies of our selected galaxies are well fitted by a density wave sweeping and compressing a large fraction of the matter in the galaxy (compare Fig. 1 with Fig. 11) which may give rise to a strong burst of star formation. So the closed box approximation models the chemical evolution well during the burst.

#### 7.2 Synthetic starlight spectrum

The synthetic spectrum of stellar populations as a function of the galactic age was derived from UV to 25  $\mu m$ . The global luminosity at the galactic age t is then obtained as the sum of the contributions of all earlier generations, weighted by the appropriate SFR as described in MXD92

The number of stars born at each galactic age t and their metallicity are obtained by solving the equations governing the chemical evolution, with the SFR and IMF specified above.

To describe their distribution in the H–R diagram we have adopted the theoretical isochrones derived by Bertelli *et al.* (1990) for metallicities Z=0.001 and Z=0.02, extended by Mazzei (1988)

up to  $100 \, m\odot$  and to an age of  $10^6$  yr. Isochrones include all evolutionary phases from the main sequence to the stage of planetary ejection or of carbon ignition, as appropriate given the initial mass.

#### 7.3 Correction for internal extinction

The internal extinction has been taken into account assuming that stars and dust are well mixed. The optical depth depends on the dust to gas ratio that we assumed to be proportional to a power of the metallicity, as in Guiderdoni and Rocca-Volmerange (1987). Further details are given in MXD92.

#### 7.4 Emission from circumstellar dust

The mid–IR emission from circumstellar dust shells was assumed to be dominated by OH/IR stars (see MXD92 for a discussion). The spectrum of OH 27.2+0.2 (Baud *et al.*, 1985) was assumed to be representative for stars of this class, then the total luminosity of OH/IR stars in the passband  $\Delta\lambda$  has been computed as in MXD92 (eq.[13]).

#### 7.5 Diffuse dust emission

The diffuse dust emission spectrum takes into account the contributions of two components: warm dust, located in regions of high radiation field intensity (e.g., in the neighborhood of OB clusters) and cold dust, heated by the general interstellar radiation field. The temperature distribution of the warm dust has been parameterized as a function of  $I_w$ , the average energy density inside HII regions. Different values of this parameter entail a some change in the mean physical conditions inside HII regions, i. e. in the neutral hydrogen density, in the  $m_l$  value, in the efficiency of star formation, and so on. The range of tested values to match the observed far–IR SED is  $20 \le I_w/I_{loc} \le 200$  where  $I_{loc}$  is the local value (see also Xu and De Zotti, 1989).

The temperature distribution of cold dust depends on the central intensity of the interstellar radiation field,  $I_0$ , and on its distribution, I(r), exponentially decreasing with the galactic radius, r. The range of tested values to match the far–IR SED is  $2 \le I_0/I_{loc} \le 45$ .

The model allows for a realistic grain—size distribution and includes PAH molecules (see Xu and De Zotti (1989) and MXD92 for more details). The amount of starlight absorbed and re—emitted by dust is determined at each time using the model for internal extinction mentioned above.

The relative contributions of the warm and cold dust components are also evolving with galactic age, the warm/cold dust ratio,  $R_{w/c}$ , being proportional to the star formation rate.

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## 10 Figure captions

- Fig. 1: R isophotes of our sample of ring galaxies.
- **Fig. 2a:** shows the time evolution of an edge—on system generated by an intrusion with an impact velocity of  $160 \, km \, s^{-1}$ . The radius of the intruder, a King sphere, is 0.28 of the target disk. The time step between different panels is 0.074 Gyr.
- Fig. 2b: is a face—on view of the time evolution for the same system as in panel a.
- **Fig. 3:** shows the time evolution of a system generating a ring galaxy endowed with a nucleus. The time step between different panels is 0.03 Gyr.
- **Fig. 4:** shows the time evolution of a system generating an empty ring galaxy. The time step between different panels is 0.03 Gyr.
- **Fig. 5:** panels a), b) and c) show the time evolution of rotation velocity and its polar and radial component, respectively, for the same model as in Fig. 2 (upper panels). The time step between different panels is 0.074 Gyr.
- **Fig. 6:** presents the behaviour of the surface density (averaged on concentric anuli) of the simulated system at different times; panel a): intrusion within the first regime of velocities; panel b): intrusion within the second regime.
- Fig. 7: shows the time evolution of the gas fraction,  $f_g$ , gas metallicity,  $Z/Z_{\odot}$ , and optical depth,  $\tau/\tau_{disk}$ , where  $Z_{\odot}$  is the solar metallicity and  $\tau_{disk}$  the effective optical depth for a pure disk (MXD92), for models computed with different  $m_l$  values and with a burst of intensity i = 60 and i = 300 lasting 0.35 and 0.20 Gyr respectively.
- **Fig. 8:** shows the time evolution of the observed colors, B-V and V-K for the same models as in Fig. 7. During the burst the time step for the plot is 0.01 Gyr.
- Fig. 9: pictures (a-e) compare the predicted SEDs with observational data (filled squares) for VV789, VV330, VV787, VV32 and NGC 2973 respectively. Heavy lines correspond to " $I_{r_0}$  case", light lines to " $I_0$  one" (see text); in picture (f) the former SEDs, long-short dashed line for VV32, short-dashed line for VV787, dotted short-dashed VV330, long-dashed VV789, dotted long-dashed NGC 2973, are compared with M82 data, dots (UBV fluxes come from RC3 catalogue, the other ones from Klein et al. 1988). Curves are normalized to B band.
- Fig. 10: pictures (a-e) compare the SEDs matching the burst (heavy lines) with those expected at an age of 14 Gyr (light lines); heavy curves refer to " $I_{r_0}$  case". The postburst evolution (see text) has been computed using  $R_{w/c} = 0.43$ ,  $I_0 = 4I_{loc}$  for VV789, VV330, VV787,  $I_0 = 7I_{loc}$  for VV32 and  $I_0 = 2I_{loc}$  for NGC 2973; the length of the burst is 0.35 Gyr with the exception of VV32 where a burst lasting 0.2 Gyr has been assumed, and VV 787 with 0.15 Gyr.

In picture (f) the postburst SEDs (symbols are as in Fig. 9) are compared with that of our galaxy (dotted line) at the same age (MXD92). Curves are normalized to B band.

Fig. 11: panel a) shows our morphological fit for VV789, the isodensity contours are taken to a stage  $\tau=0.20$  from the beginning of the ring structure; panel b) refers to VV330 with  $\tau=0.40$ ; panel c): VV32, t=0.044 Gyr from the beginning of the ring structure; panel d): NGC 2793,  $\tau=0.17$